# CSE 590K: Analysis and Control of Computing Systems Using Linear Discrete-Time System Theory:
# Common Controllers & Controller Design

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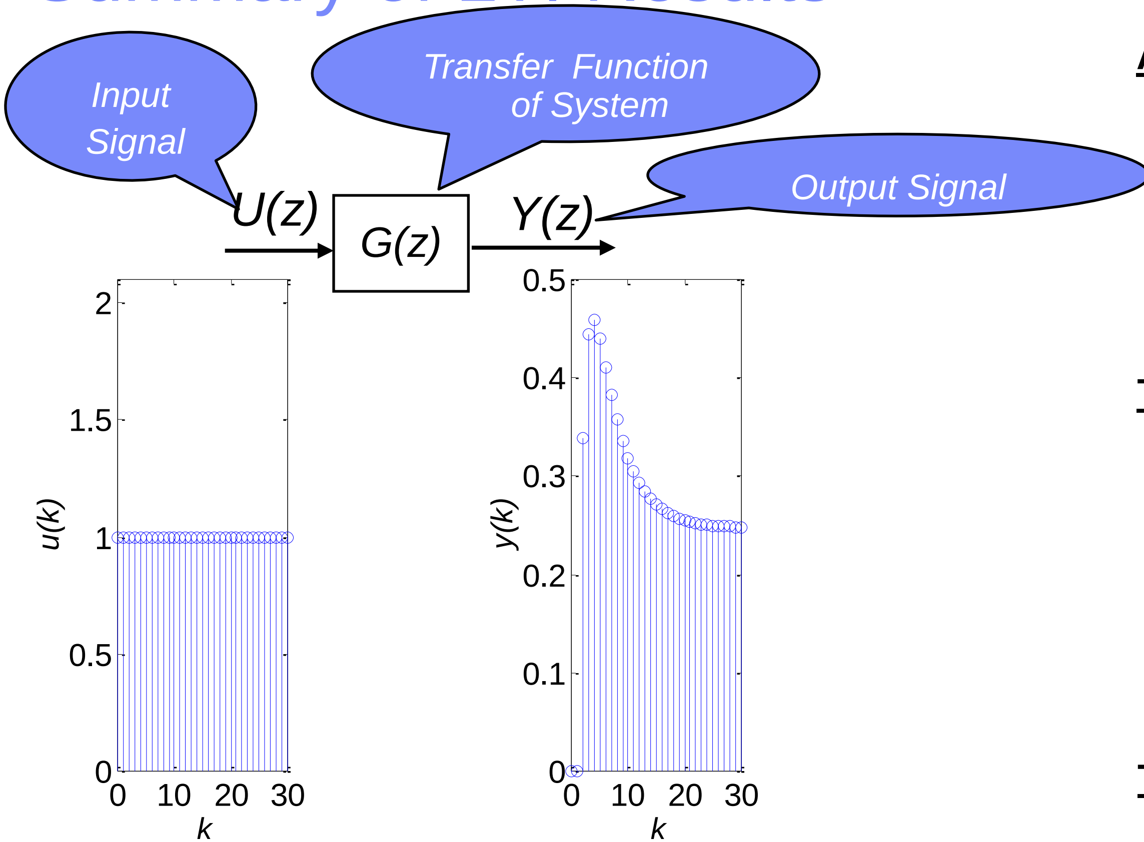
# Agenda

- Common SISO (Single Input, Single Output) Controllers
- Design of SISO control
- Lab: Control Analysis

# Summary of LTI Results

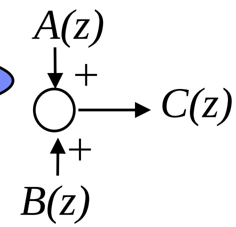



Stable system if $|a|<1$, where $a$ is the largest pole of $G(z)$

$$\text{Settling time} \approx \frac{-4}{\ln|a|}, \text{ where } |a| \text{ is the largest pole of } G(z)$$

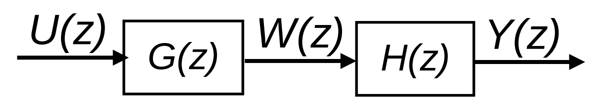
Steady state gain of $G(z)$ is $\frac{y(\infty)}{u(\infty)} = G(1)$

**Adding signals**:

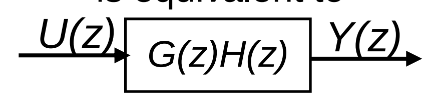



$\{c(k)=a(k)+b(k)\}$ has Z-Transform $A(z)+B(z)$.

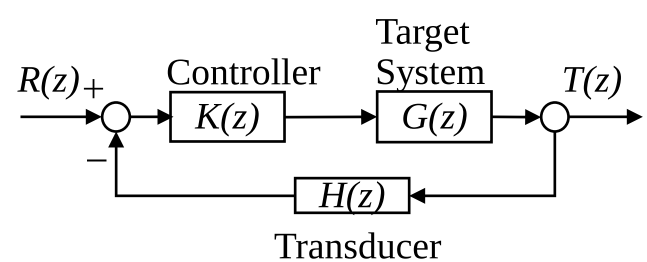
**Transfer functions in series**




is equivalent to

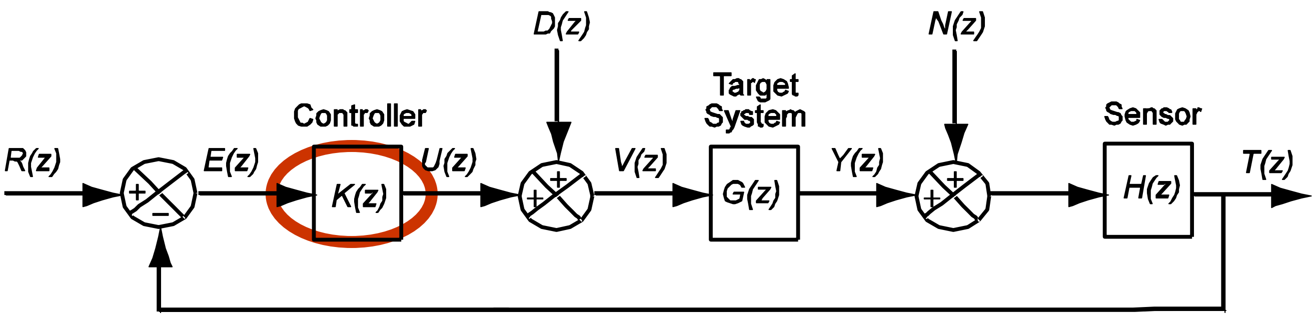



**Transfer function of a feedback loop**

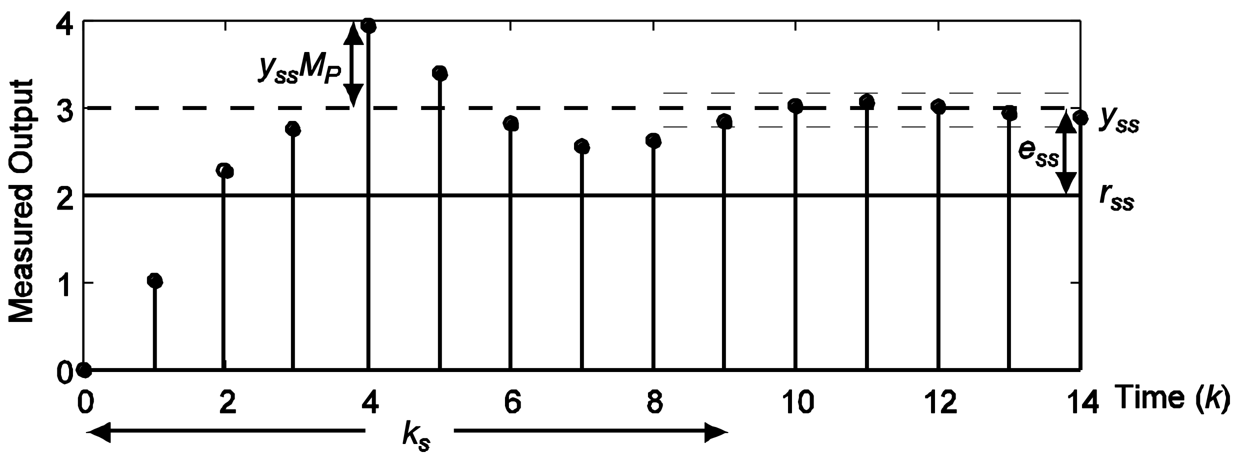



$$F_R(z) = \frac{T(z)}{R(z)} = \frac{K(z)G(z)}{1 + H(z)K(z)G(z)}$$

# Basic Controllers




**Basic controller**
- Input: *E(z)*
- Output: *U(z)*




**Given**
- Target system t.f. *G(z)*
- Transducer t.f. *H(z)*
- Controller t.f. *K(z)*

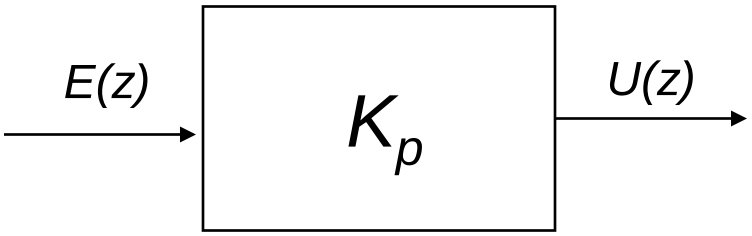
**Find**
- Stability, accuracy, settling times
  - Poles, steady state gain

# Proportional Control

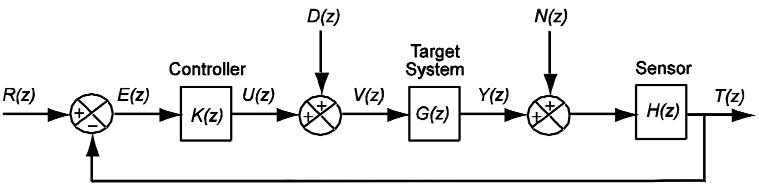
$$u(k) = K_P e(k)$$

$$K(z) = \frac{U(z)}{E(z)} = K_P$$

Control gain
(or just gain)



# Performance of Proportional Control

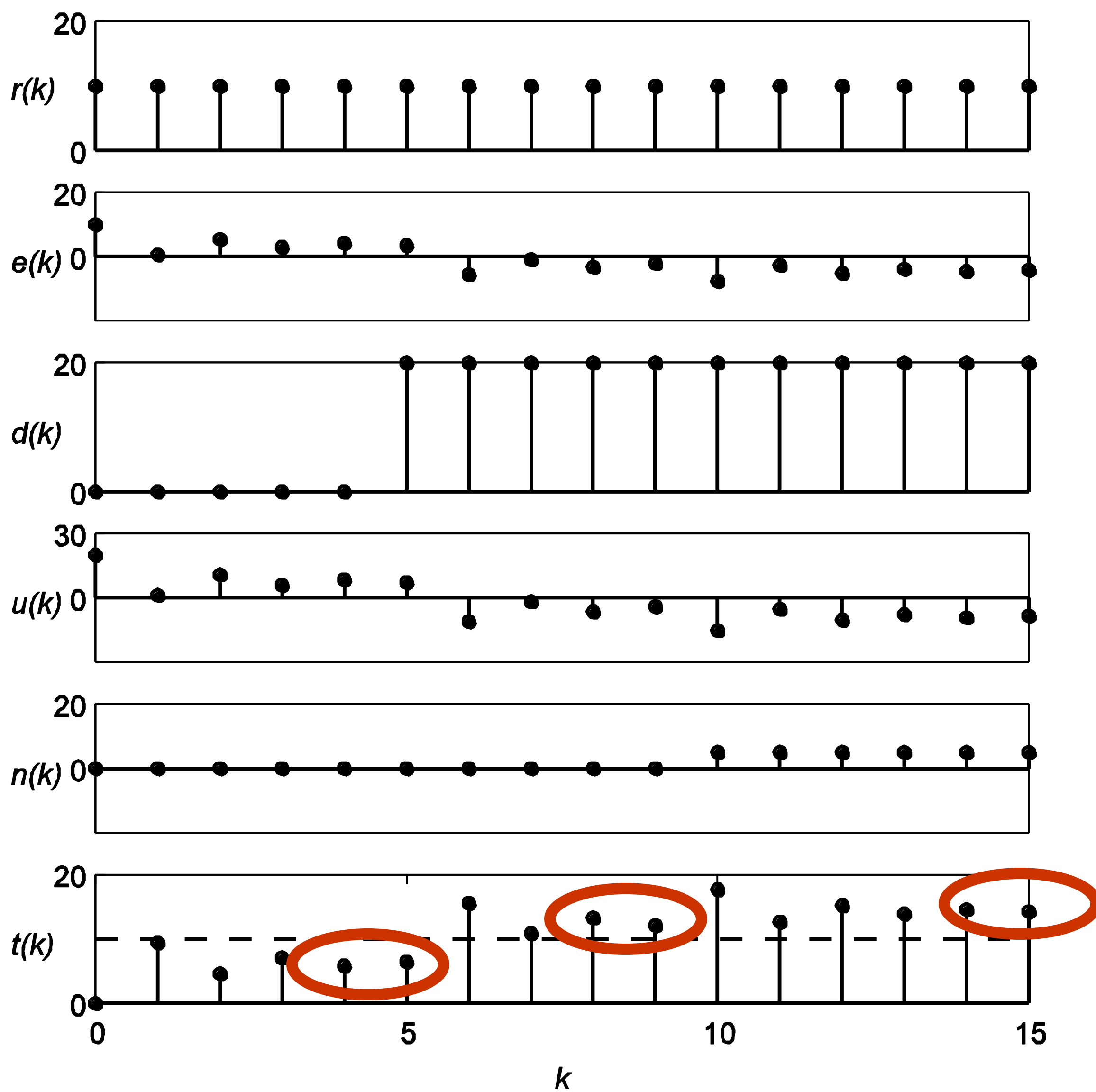



$$K(z) = K_P = 2$$

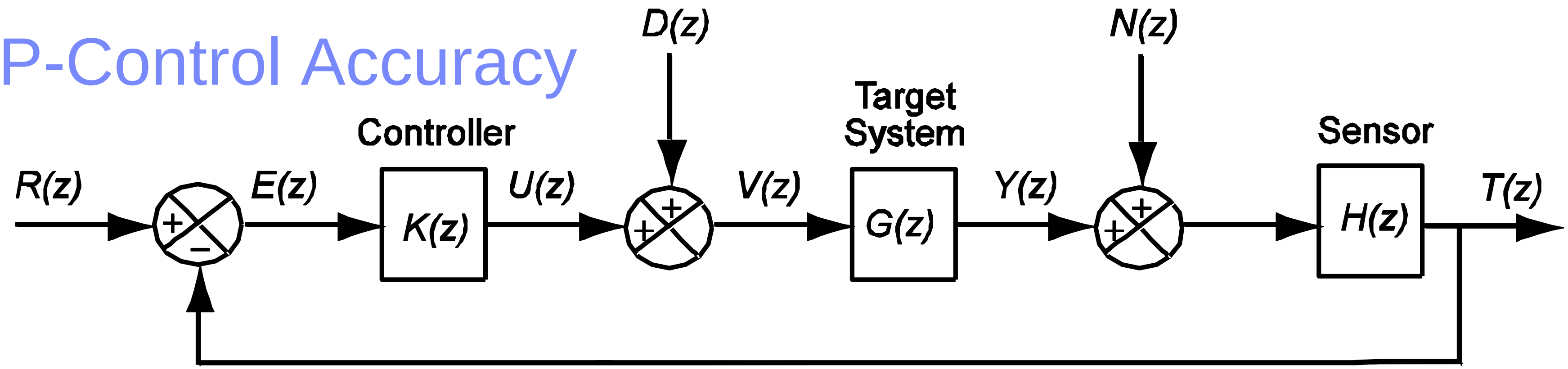
$$G(z) = \frac{0.47}{z - 0.43}$$

$$H(z) = 1$$

How does $K_P$ affect:
- Stability
- Accuracy
- Settling time



# P-Control Accuracy




$$K(z) = K_P$$

$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{K_P G(z) H(z)}{1 + K_P G(z) H(z)}$$

$$F_D(z) = \frac{Y(z)}{D(z)} = \frac{G(z) H(z)}{1 + K_P G(z) H(z)}$$

$$F_N(z) = \frac{T(z)}{N(z)} = \frac{H(z)}{1 + K_P G(z) H(z)}$$

Why?

Want:

$$F_R(1) = \frac{K_P G(1) H(1)}{1 + K_P G(1) H(1)} = 1$$

$$F_D(1) = \frac{G(1) H(1)}{1 + K_P G(1) H(1)} = 0$$

$$F_N(1) = \frac{H(1)}{1 + K_P G(1) H(1)} = 0$$

# Accuracy of Closed-loop Transfer Functions

$$G(z) = \frac{Y(z)}{U(z)} = \frac{0.47}{z - 0.43}$$

$$F_R(z) = \frac{0.47K_P}{z - 0.43 + 0.47K_P}$$

For $K_P = 2$

$$F_R(1) = \frac{0.47K_P}{0.57 + 0.47K_P} = 0.62 < 1$$

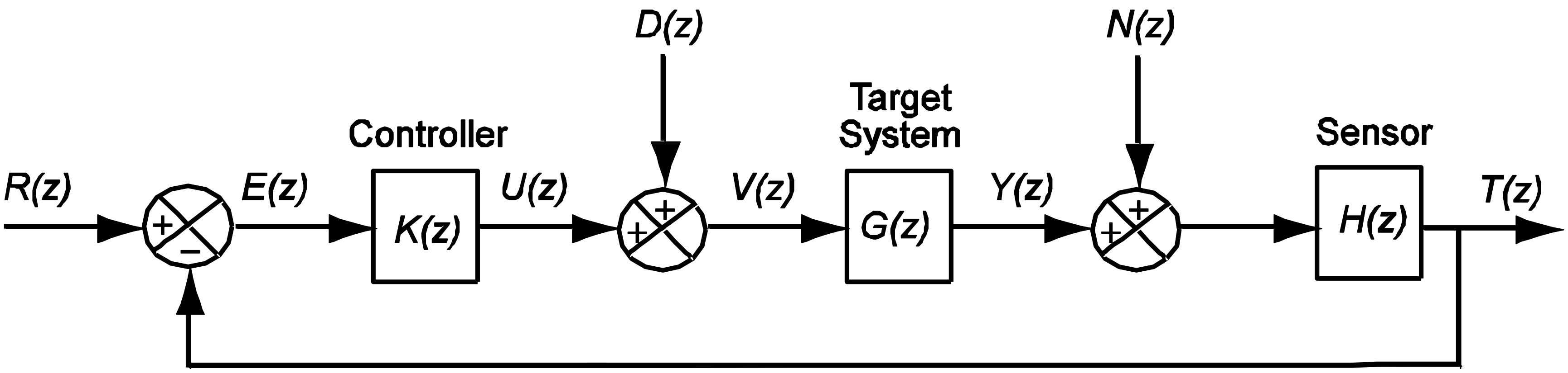
$$F_D(1) = \frac{0.47}{0.57 + 0.47K_P} = 0.31 > 0$$

$$F_N(1) = \frac{1}{0.57 + 0.47K_P} = 0.66 > 0$$

**Observations**

- Inherent inaccuracy of P-control
- Want large $K_P$

# P-Control Stability and Settling Times




$$K(z) = K_P$$

$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{K_P G(z) H(z)}{1 + K_P G(z) H(z)}$$

$$F_D(z) = \frac{Y(z)}{D(z)} = \frac{G(z) H(z)}{1 + K_P G(z) H(z)}$$

$$F_N(z) = \frac{T(z)}{N(z)} = \frac{H(z)}{1 + K_P G(z) H(z)}$$

**Observation**:
•Transfer functions have the same poles

Defn: ***Characteristic polynomial***
•Denominator of the transfer function
Defn: ***Characteristic equation***
•Set characteristic polynomial to 0

# Poles of Closed-loop Transfer Functions

$$G(z)=\frac{Y(z)}{U(z)}=\frac{0.47}{z-0.43},\ H(z)=1$$

$$F_R(z)=\frac{0.47K_P}{z-0.43+0.47K_P}$$

Why complex?

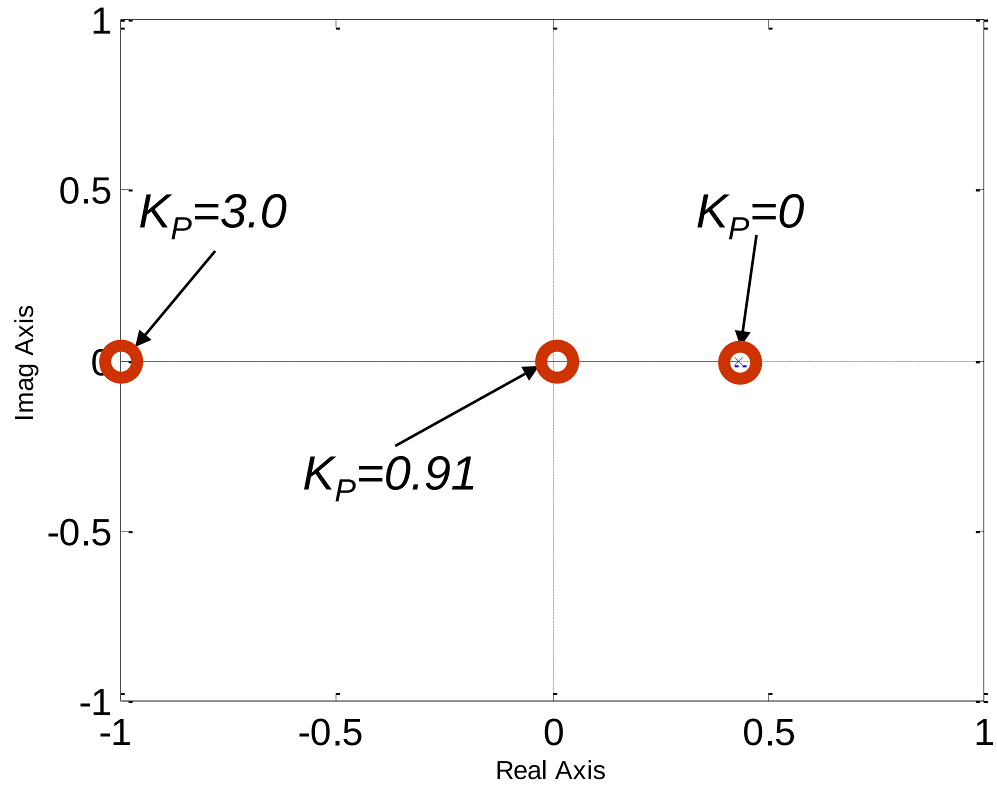
Characteristic equation:

$$1+K_P G(z)H(z)=0$$

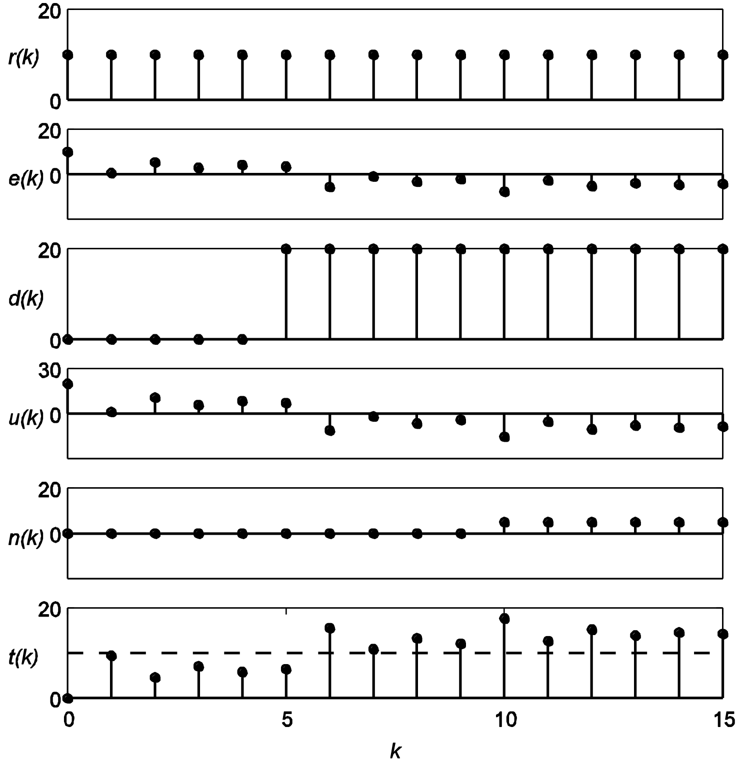
Poles are solutions to characteristic equation.

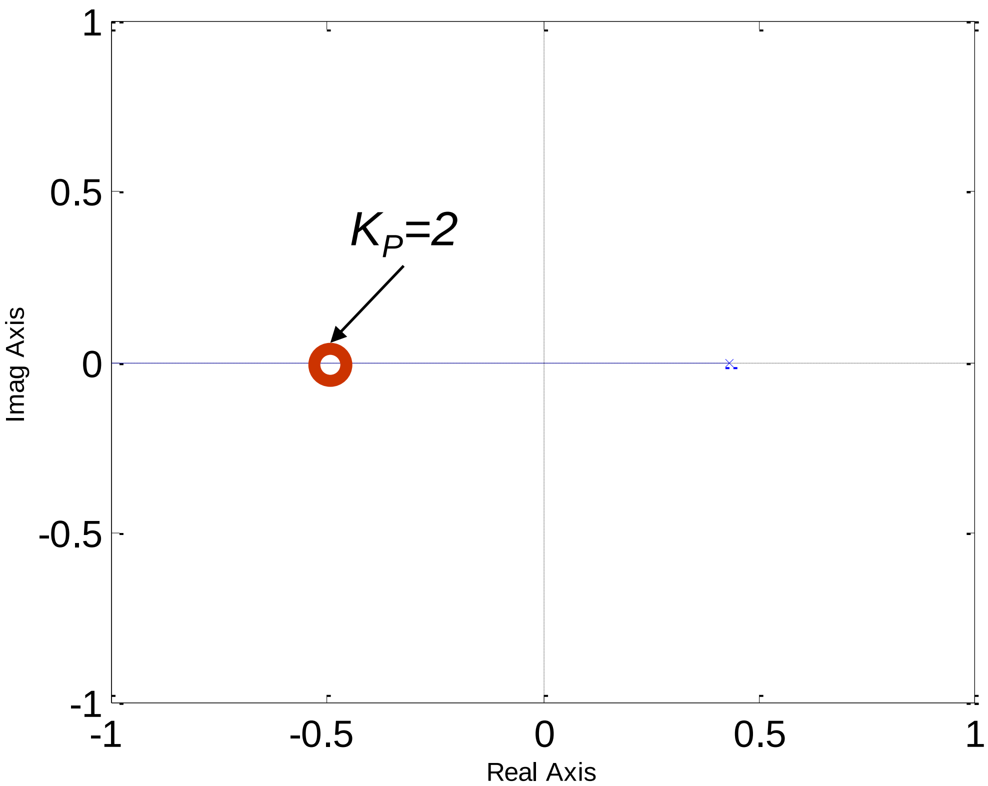
**Root locus**
- Plot $K_P$ that solve the characteristics equation

Plot $z=0.43-0.47K_P$ in the complex plane



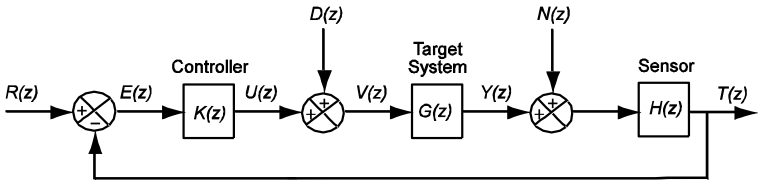
# Explaining the Simulation Settling Times With Root-Locus




At $K_P = 2,\ z = 0.43 - 0.47(2) = -0.51.\ k_S \approx 6.$

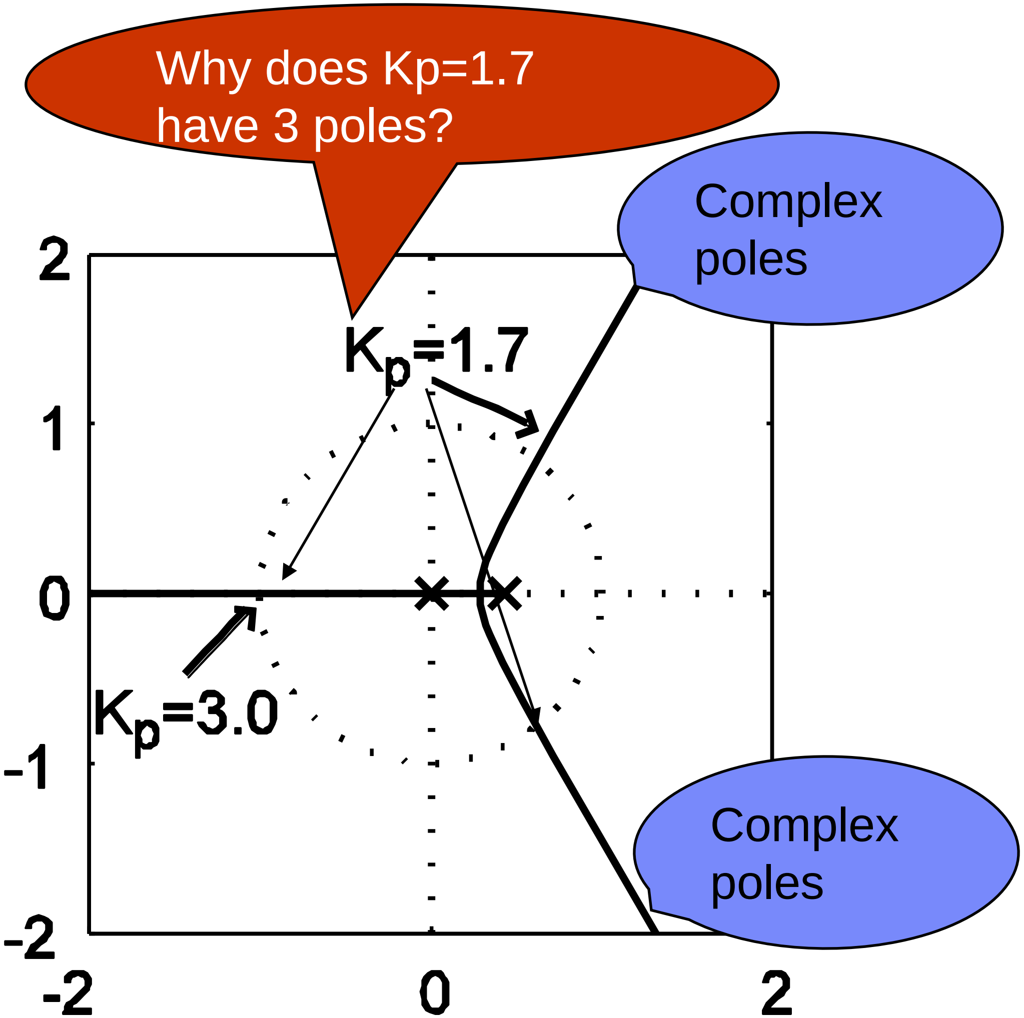
# More Complicated Root-locus Plots




$$G(z) = \frac{Y(z)}{U(z)} = \frac{0.47}{z - 0.43}$$

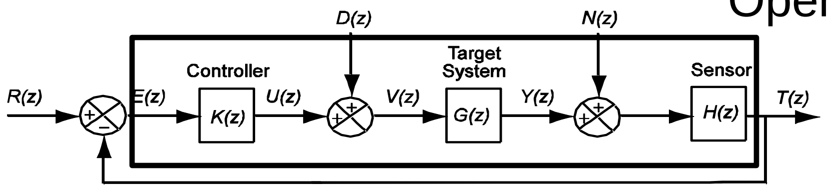
$$H(z) = \frac{1}{z^2} \quad \text{(delay of two time units)}$$

$$F_R(z) = \frac{0.47K_P}{z^3 - z^2 0.43 + 0.47K_P}$$




Why two branches for complex poles?

# Towards Constructing Root-Locus Plots




Open loop. Open loop transfer function: *G'(z)*

$$K(z) = KK'(z)$$
$$G'(z) = K'(z)G(z)H(z)$$

$$F_R(z) = \frac{T(z)}{R(z)} = \frac{K(z)G(z)H(z)}{1+K(z)G(z)H(z)} = \frac{KG'(z)}{1+KG'(z)}$$

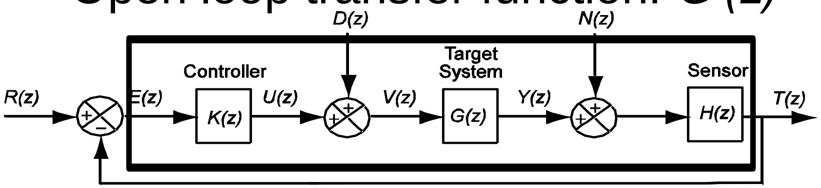
Poles are $z$ solutions to: $1 + KG'(z) = 0$

$$\Leftrightarrow \frac{1}{K} + G'(z) = 0. \text{ At } K = \infty, \text{ poles of } G'(z).$$

$$\Leftrightarrow \frac{1}{G'(z)} + K = 0. \text{ At } K = 0, \text{ zeroes of } G'(z).$$

# Outline for Constructing Root-Locus Plots

Open loop transfer function: *G’(z)*

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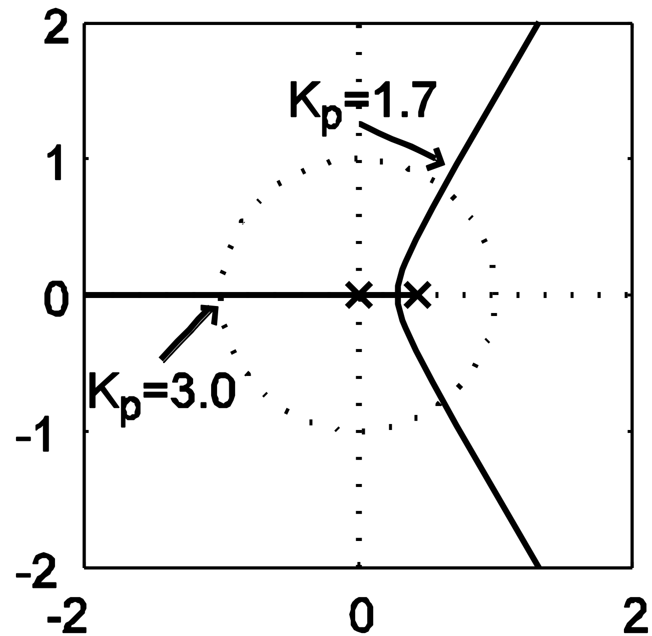
$$\Leftrightarrow \frac{1}{G'(z)} + K = 0. \text{ At } K = 0, \text{ zeroes of } G'(z).$$



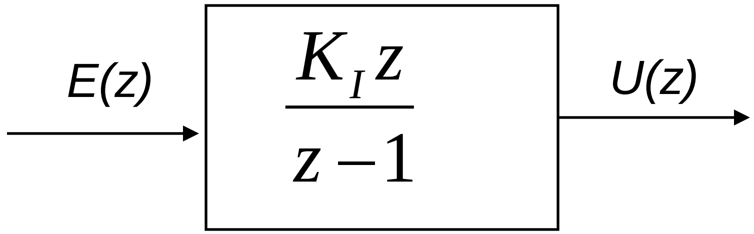

**Steps**

1. Indicate open loop poles with an “x”
2. Indicate open loop zeros with an “o”
3. Determine the root loci on the real axis
4. Find the break-away and break-in points
5. Draw the line in the complex plane that intersects the real axis

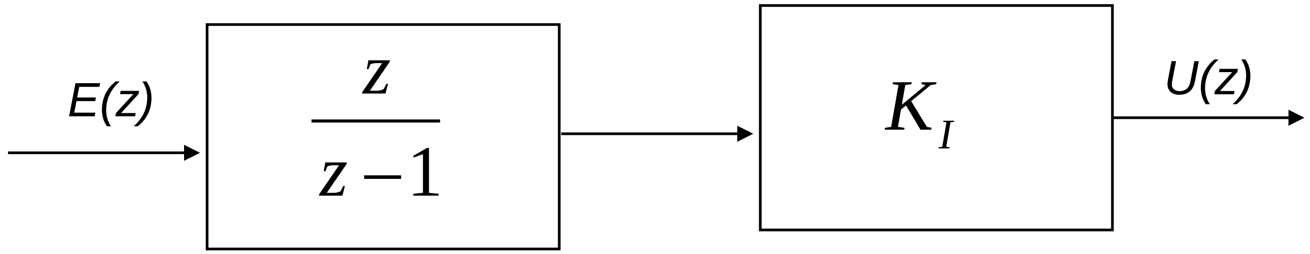
# Integral Control

$$u(k+1) = K_I u(k) + e(k+1)$$

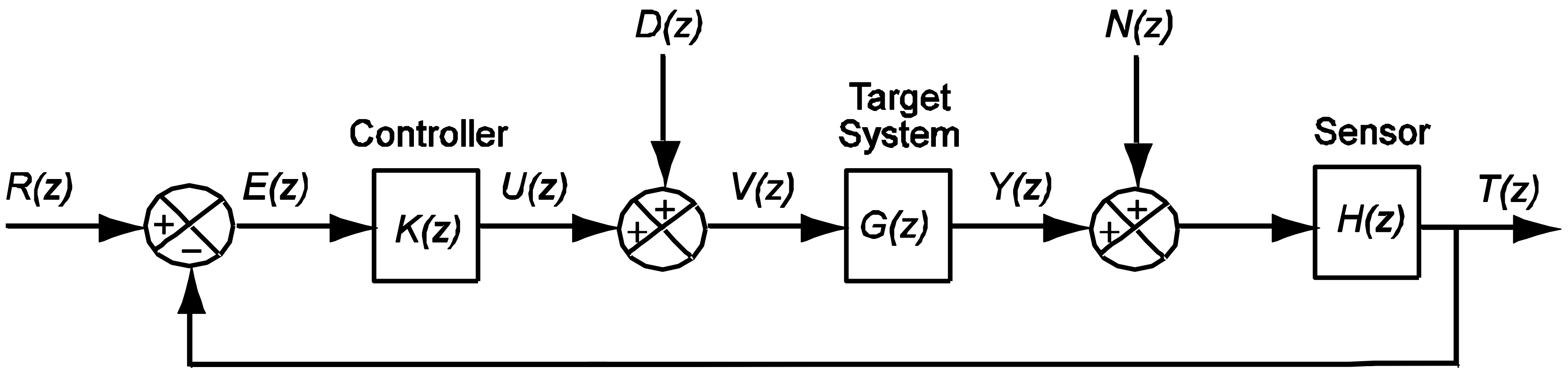
$$K(z) = \frac{U(z)}{E(z)} = \frac{K_I z}{z-1}$$




Or



# I-Control Accuracy




$$K(z) = \frac{U(z)}{E(z)} = \frac{K_I z}{z-1}, \; H(z) = 1$$

$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{zK_I G(z)}{z-1+zK_I G(z)}$$

$$F_D(z) = \frac{Y(z)}{D(z)} = \frac{G(z)(z-1)}{z-1+zK_I G(z)}$$

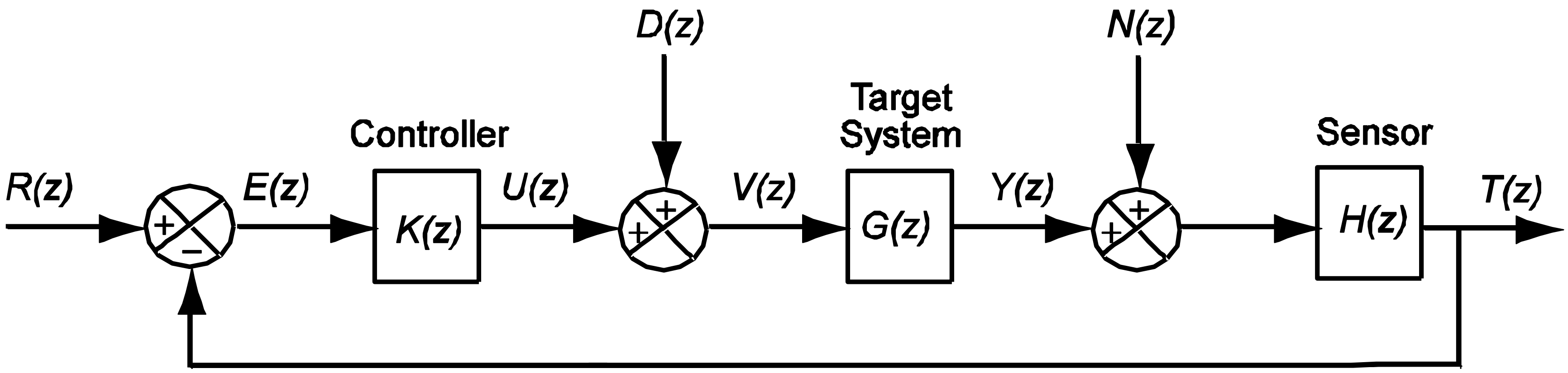
$$F_N(z) = \frac{T(z)}{N(z)} = \frac{(z-1)}{z-1+zK_I G(z)}$$

Observe that

$$F_R(1) = \frac{K_I G(1)}{1-1+K_I G(1)} = 1$$

$$F_D(1) = 0 = F_N(1)$$

# I-Control Stability and Settling Times




$$K(z) = \frac{U(z)}{E(z)} = \frac{K_I z}{z-1}$$

$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{zK_I G(z)H(z)}{z-1+zK_I G(z)H(z)}$$

$$F_D(z) = \frac{Y(z)}{D(z)} = \frac{G(z)H(z)}{z-1+zK_I G(z)H(z)}$$

$$F_N(z) = \frac{T(z)}{N(z)} = \frac{H(z)}{z-1+zK_I G(z)H(z)}$$

I-Control characteristic equation:

$$0 = z-1+zK_I G(z)H(z)$$

P-Control characteristic equation:

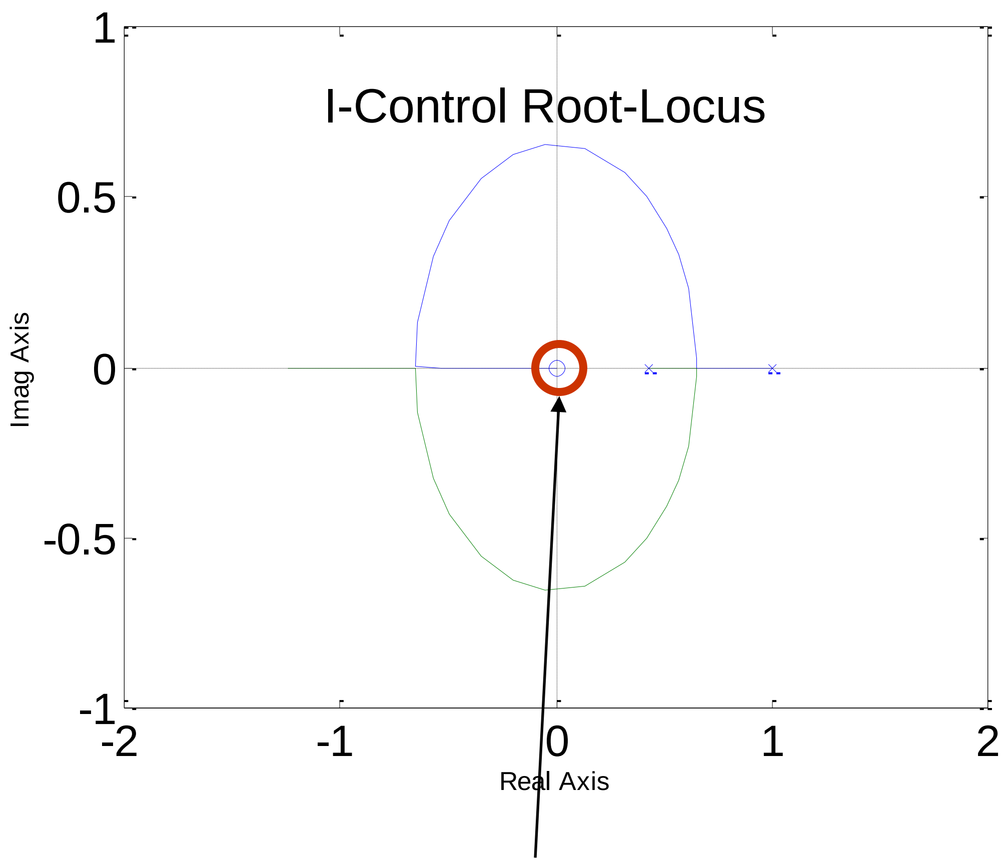
$$0 = 1+K_P G(z)H(z)$$

I-Control results in a higher order system.

# Root Locus for I-Control: No Feedback Delay

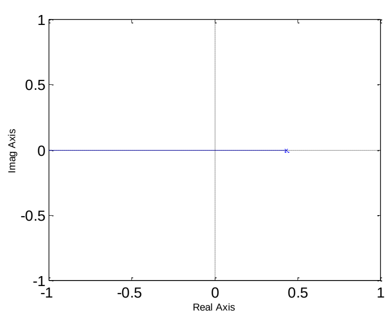
$$G(z)=\frac{Y(z)}{U(z)}=\frac{0.47}{z-0.43},\ H(z)=1$$

$$F_R(z)=\frac{0.47K_I z}{z^2+(0.47K_I-1.43)z+0.43}$$

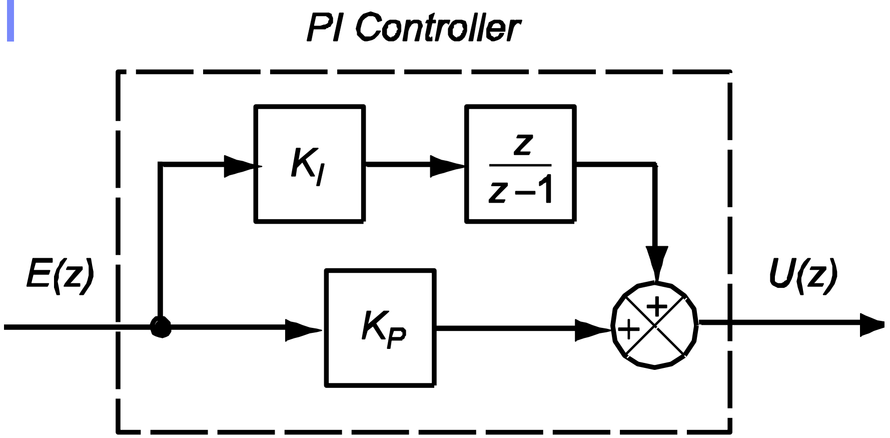



Why the open-loop zero at 0?

P-Control Root-Locus



# PI Control

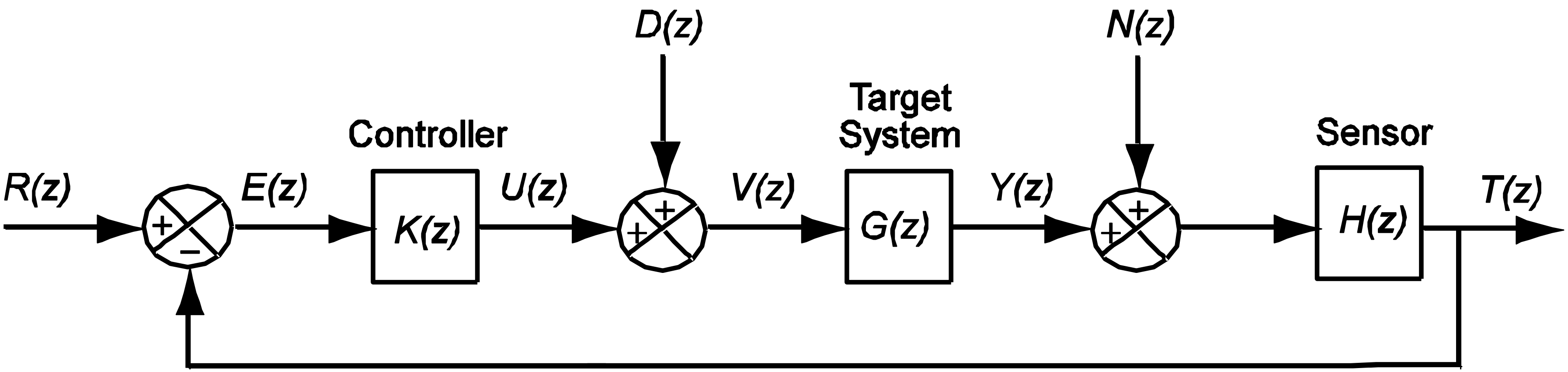



$$u(k) = u_P(k) + u_I(k)$$

$$= u(k-1) + (K_P + K_I)e(k) - K_P e(k-1)$$

$$K(z) = \frac{E(z)}{U(z)} = K_P + \frac{K_I z}{z-1} = \frac{(K_P + K_I)z - K_P}{z-1}$$

# PI Control Accuracy




$$K(z) = \frac{U(z)}{E(z)} = \frac{(K_P + K_I)z - K_P}{z-1}, \; H(z) = 1$$

$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{((K_P + K_I)z - K_P)G(z)}{z - 1 + ((K_P + K_I)z - K_P)G(z)}$$

$$F_D(z) = \frac{Y(z)}{D(z)} = \frac{G(z)(z-1)}{z - 1 + ((K_P + K_I)z - K_P)G(z)}$$

$$F_N(z) = \frac{T(z)}{N(z)} = \frac{(z-1)}{z - 1 + ((K_P + K_I)z - K_P)G(z)}$$

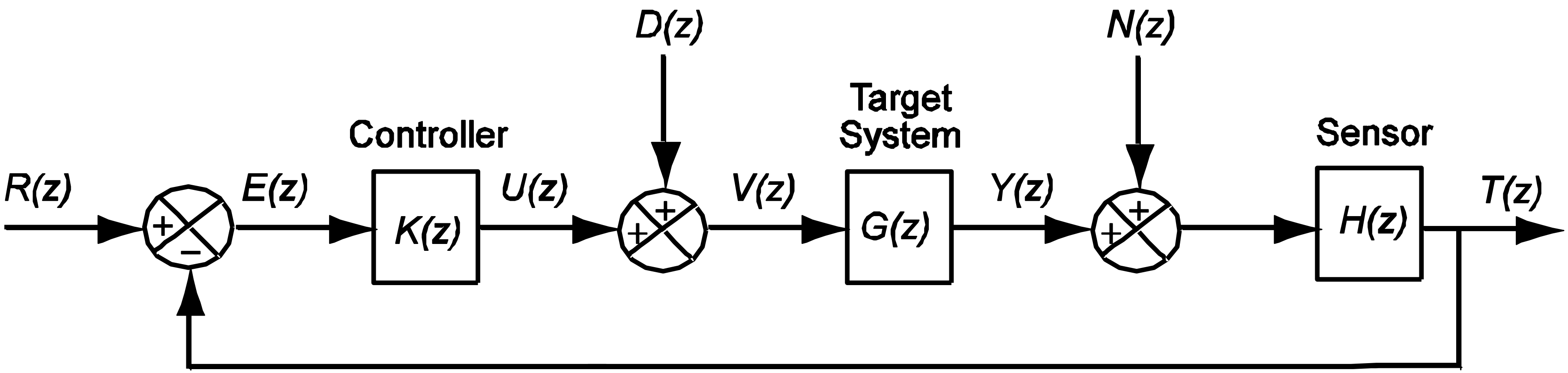
Observe that

$$F_R(1) = 1$$

$$F_D(1) = 0 = F_N(1)$$

Integral component ensures accuracy for a step response

# PI Control Stability and Settling Times




$$K(z) = \frac{U(z)}{E(z)} = \frac{(K_P + K_I)z - K_P}{z-1}, \ H(z) = 1$$

$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{((K_P + K_I)z - K_P)G(z)}{z - 1 + ((K_P + K_I)z - K_P)G(z)}$$

$$F_D(z) = \frac{Y(z)}{D(z)} = \frac{G(z)}{z - 1 + ((K_P + K_I)z - K_P)G(z)}$$

$$F_N(z) = \frac{T(z)}{N(z)} = \frac{1}{z - 1 + ((K_P + K_I)z - K_P)G(z)}$$

I Control characteristic equation:

$$0 = z - 1 + zK_I G(z)H(z)$$

P Control characteristic equation:

$$0 = 1 + K_P G(z)H(z)$$

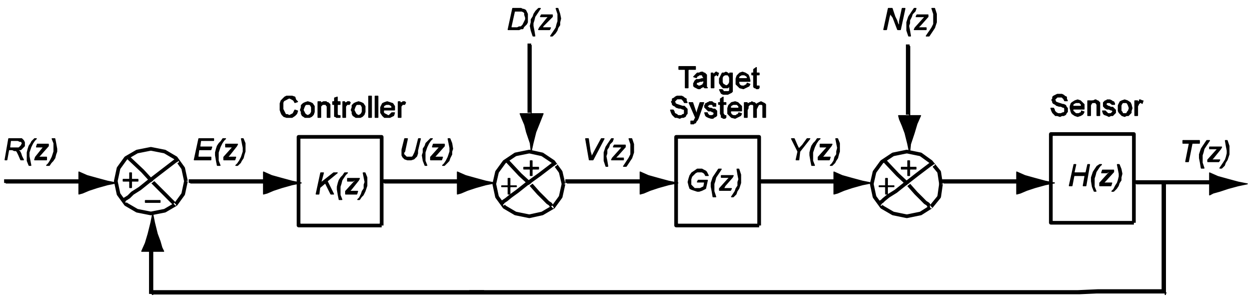
PI Control characteristic equation:

$$0 = z - 1 + ((K_P + K_I)z - K_P)G(z)$$

# Root Locus with > 1 Control Gain

- Issues
  - Two parameters
  - Transfer function is not in a convenient form for root locus
- Approach
  - Look at the largest pole (and possible pole angle)
  - Can translate into settling time

# PI Poles, Stability, and Settling Times: No Sensor Delays

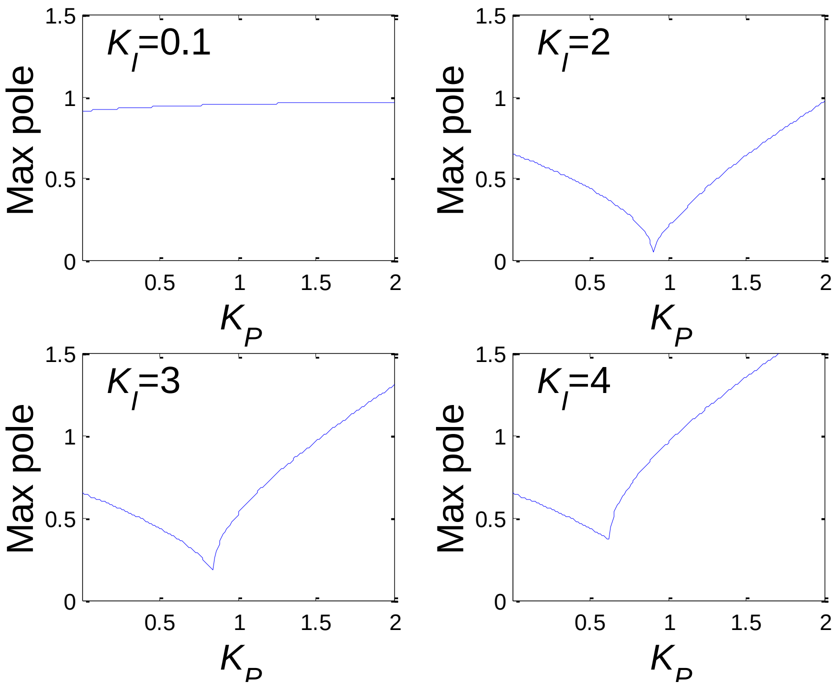


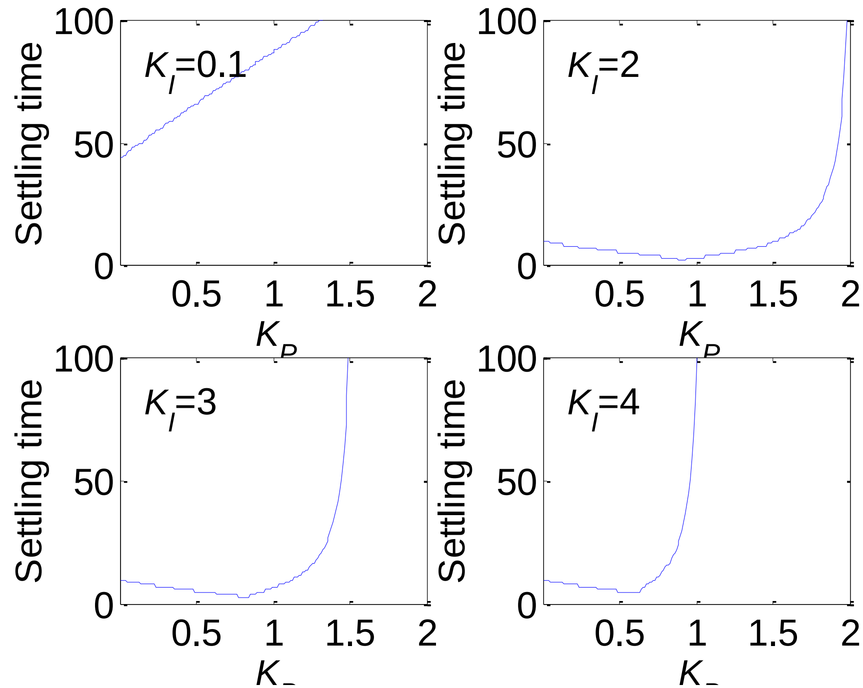

$$K(z) = \frac{U(z)}{E(z)} = \frac{(K_P + K_I)z - K_P}{z-1}$$

$$G(z) = \frac{Y(z)}{U(z)} = \frac{0.47}{z-0.43}$$

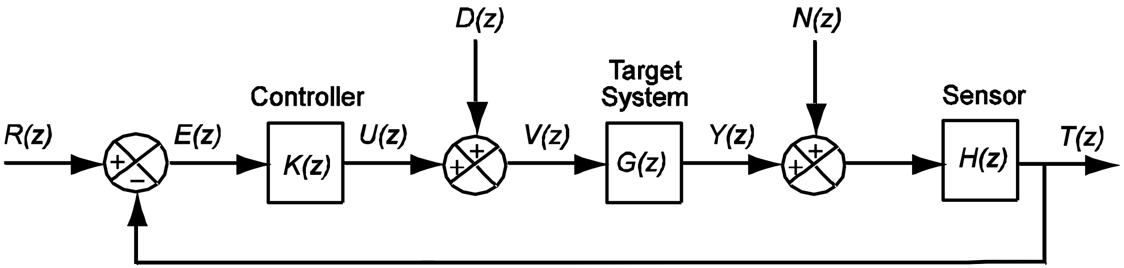
$$H(z) = 1$$

$$F_R(z) = \frac{0.47(K_P + K_I)z - 0.47K_P}{z^2 + \left(0.47(K_P + K_I) - 1.43\right)z + 0.43 - 0.47K_P}$$



# PI Poles, Stability, and Settling Times: With Sensor Delays

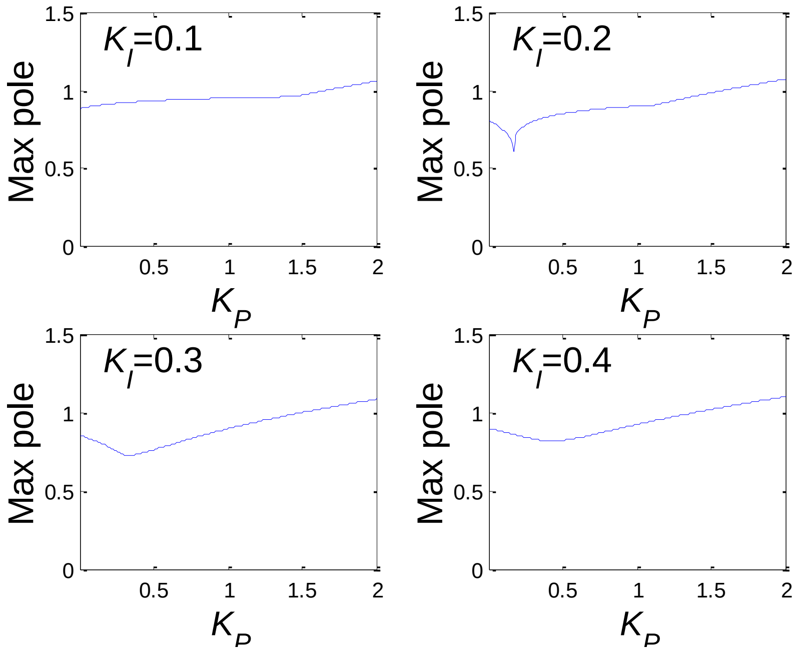


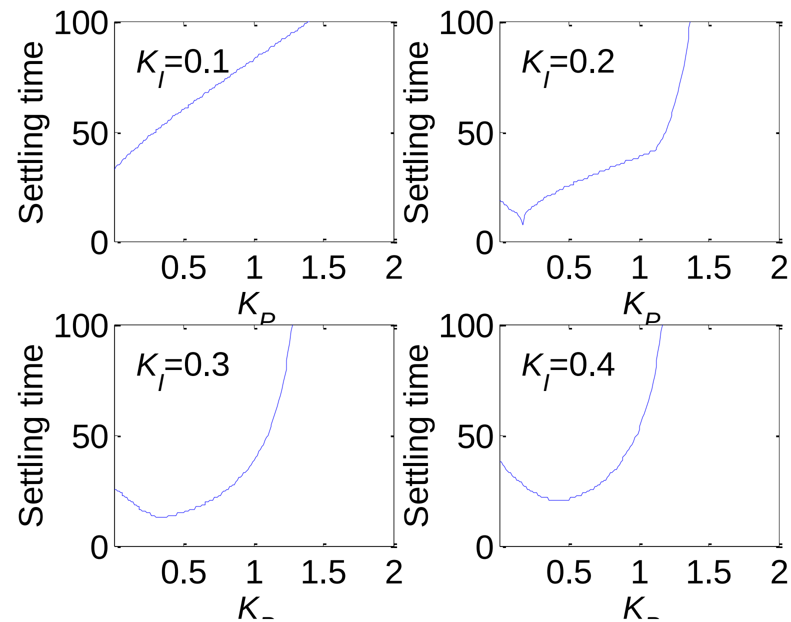

$$K(z) = \frac{U(z)}{E(z)} = \frac{(K_P + K_I)z - K_P}{z-1}$$

$$G(z) = \frac{Y(z)}{U(z)} = \frac{0.47}{z-0.43}$$

$$H(z) = \frac{1}{z^2}$$

$$F_R(z) = \frac{0.47(K_P + K_I)z - 0.47K_P}{z^4 - 1.43z^3 + 0.43z^2 + 0.47(K_P + K_I)z - 0.47K_P}$$



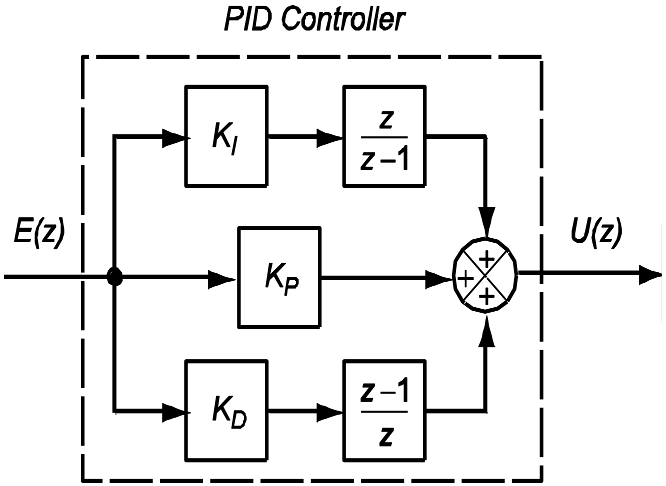
# Differential Control

$$u_D(k) = K_D(e(k) - e(k-1))$$

$$K_D(z) = \frac{z-1}{z}$$

Always used in combination with Integral and proportional control
Appeal: Anticipate trends

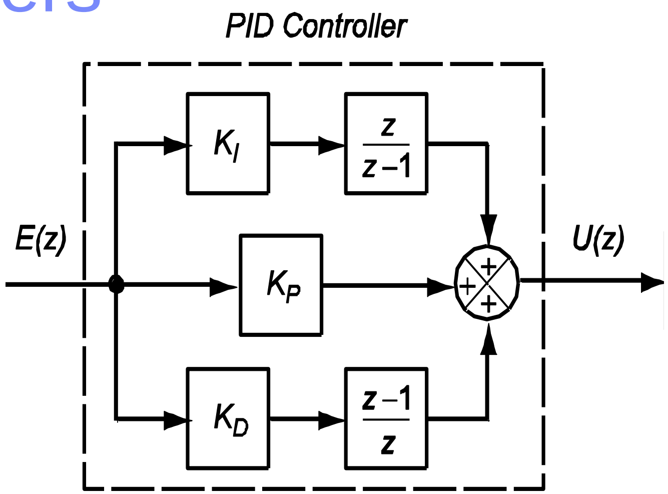
# PID Control




$$
\begin{aligned}
u(k) &= u_P(k) + u_I(k) + u_D(k) \\
&= K_P e(k) + u(k-1) + K_I e(k) + K_D(e(k) - e(k-1))
\end{aligned}
$$

$$
K(z) = \frac{E(z)}{U(z)} = K_P + K_I \frac{z}{z-1} + K_D \frac{z-1}{z}
$$

# Basic Controllers




P control $$K(z) = K_P \qquad u(k) = K_P e(k)$$

I control $$K(z) = K_I \frac{z}{z-1} \qquad u(k) = u(k-1) + K_I e(k)$$

D control $$K(z) = K_D \frac{z-1}{z} \qquad u(k) = K_D[e(k) - e(k-1)]$$

PI control $$K(z) = K_P + K_I \frac{z}{z-1} \qquad u(k) = u(k-1) + K_P[e(k) - e(k-1)] + K_I e(k)$$

PID control $$K(z) = K_P + K_I \frac{z}{z-1} + K_D \frac{z-1}{z} \qquad u(k) = \begin{cases} u(k-1) + K_P[e(k) - e(k-1)] + K_I e(k) \\ + K_D[e(k) - 2e(k-1) - e(k-2)] \end{cases}$$

# Controller Design

# Intuition For Designing Controllers

- Controllers are parameterized by their gain constants
  - $K_P$, $K_I$, $K_D$
- Gains determine the closed-loop poles

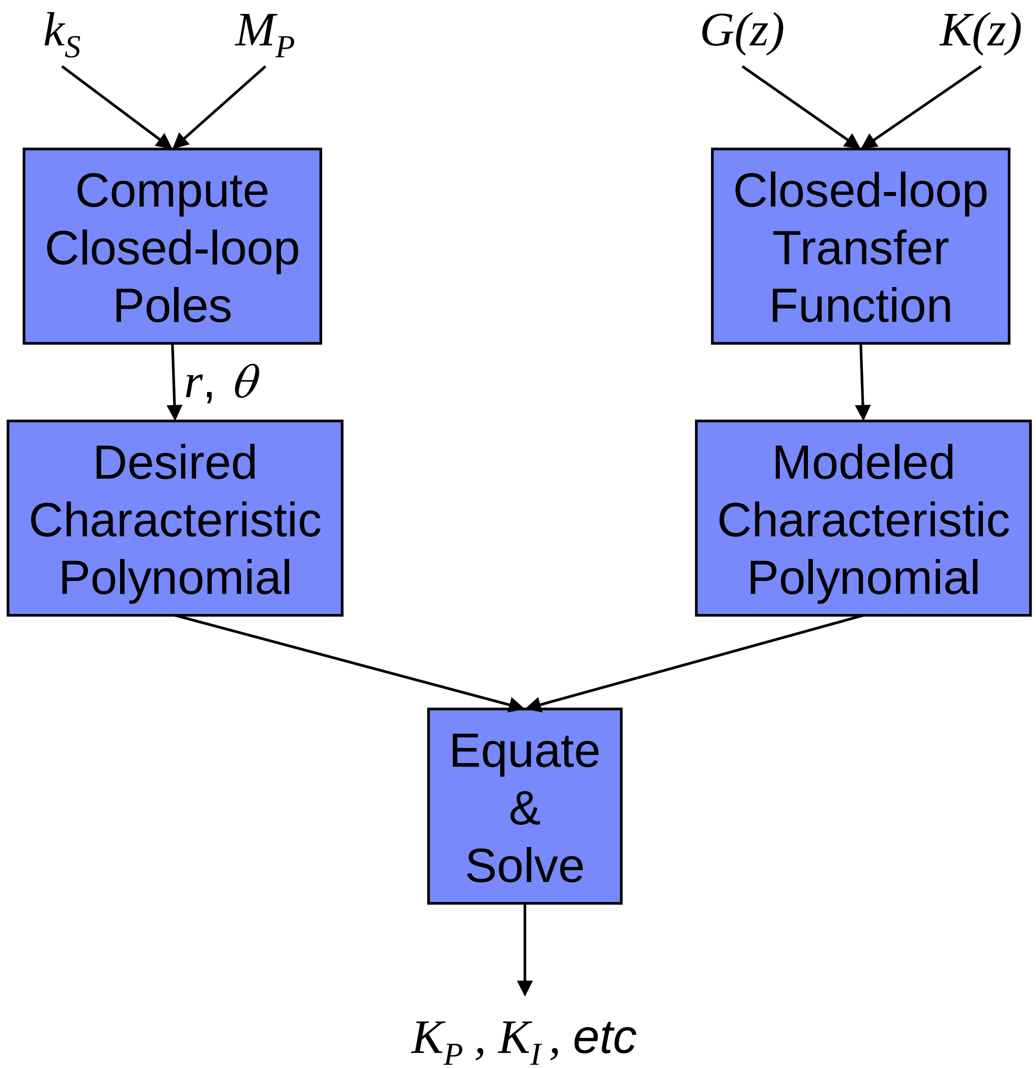
$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{K(z)G(z)H(z)}{1 + K(z)G(z)H(z)}$$

- Thus, system response is determined by controller parameters
  - The effect can be determined from the characteristic equation

# Controller Design Problem

- Given
  - System Model
    - Or, 1st-order approximation
  - Controller
  - Control Objective
    - Reference Tracking
    - Disturbance Rejection
  - Desired Transient response characteristics
    - Settling Time constraint
    - Overshoot constraint
    - Steady-state error [?]
- Choose
  - Controller gains

# Pole Placement: General Methodology



# Pole placement for I-control of Notes

Desired Characteristic Polynomial:

$$p_{1,2} = re^{\pm j\theta}$$

$$k_S \approx -\frac{4}{\log r} \quad \Rightarrow \quad r = e^{-4/k_s}$$

$$M_P \approx r^{\pi/\theta} \quad \Rightarrow \quad \theta = \pi \frac{\log r}{\log M_P}$$

$$P_D(z) = (z - re^{j\theta})(z - re^{-j\theta}) = z^2 - (2r\cos\theta)z + r^2$$

Modeled Characteristic Polynomial:

$$G(z) = \frac{0.47}{z - 0.43}, \quad K(z) = \frac{K_I z}{z - 1}$$

$$F_R(z) = \frac{0.47 K_I z}{z^2 + (0.47K_I - 1.43)z + 0.43}$$

$$P_M(z) = z^2 + (0.47K_I - 1.43)z + 0.43$$

$$P_D(z) = P_M(z)$$

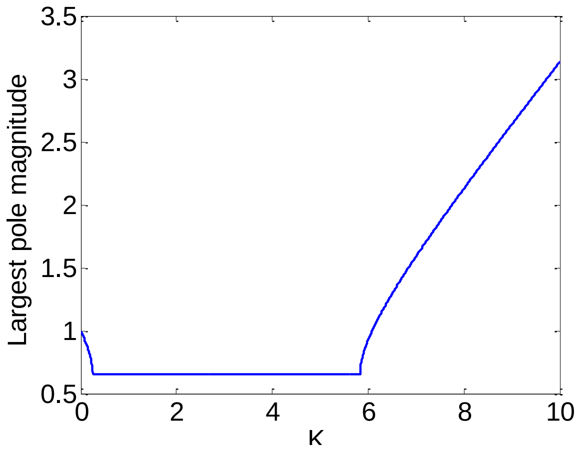
$$r^2 = 0.43$$

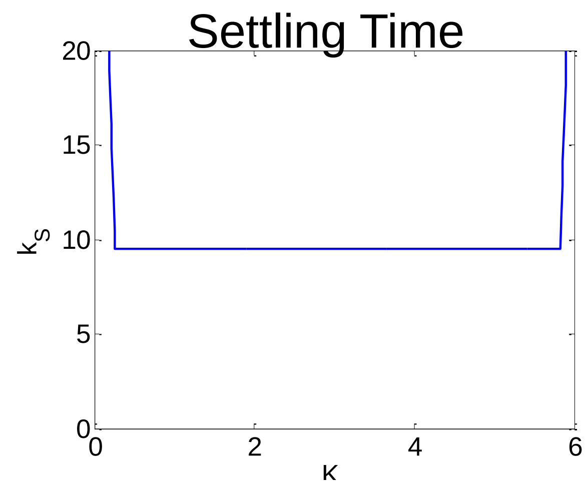
$$-2r\cos\theta = (0.47K_I - 1.43)$$

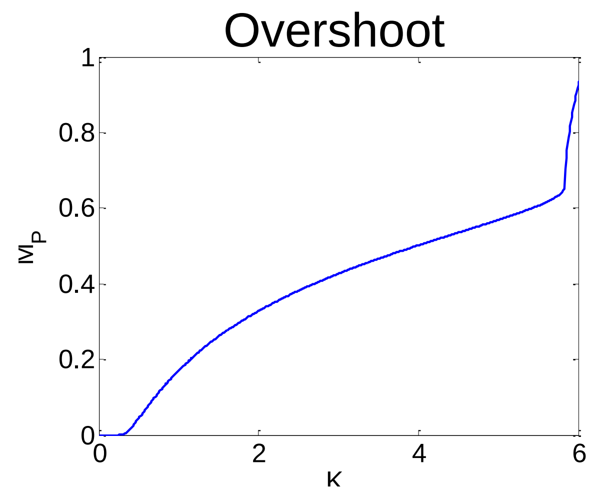
PI control allows selecting settling time and overshoot

- Assume COMPLEX poles $\Rightarrow$ *Cannot* choose $k_S$.
- Can repeat analysis for REAL poles.
- $K_I$ only affects $M_P$.

# Effect of $K_I$ on system dynamics



# Empirical Design: CHR Heuristic

- Empirical Design Techniques
  - CHR
    - Bump-test for system ID
    - Approximate high-order system by combination of:
      - Pure delay
      - First-order system
    - Choose K_P, K_I based on test results

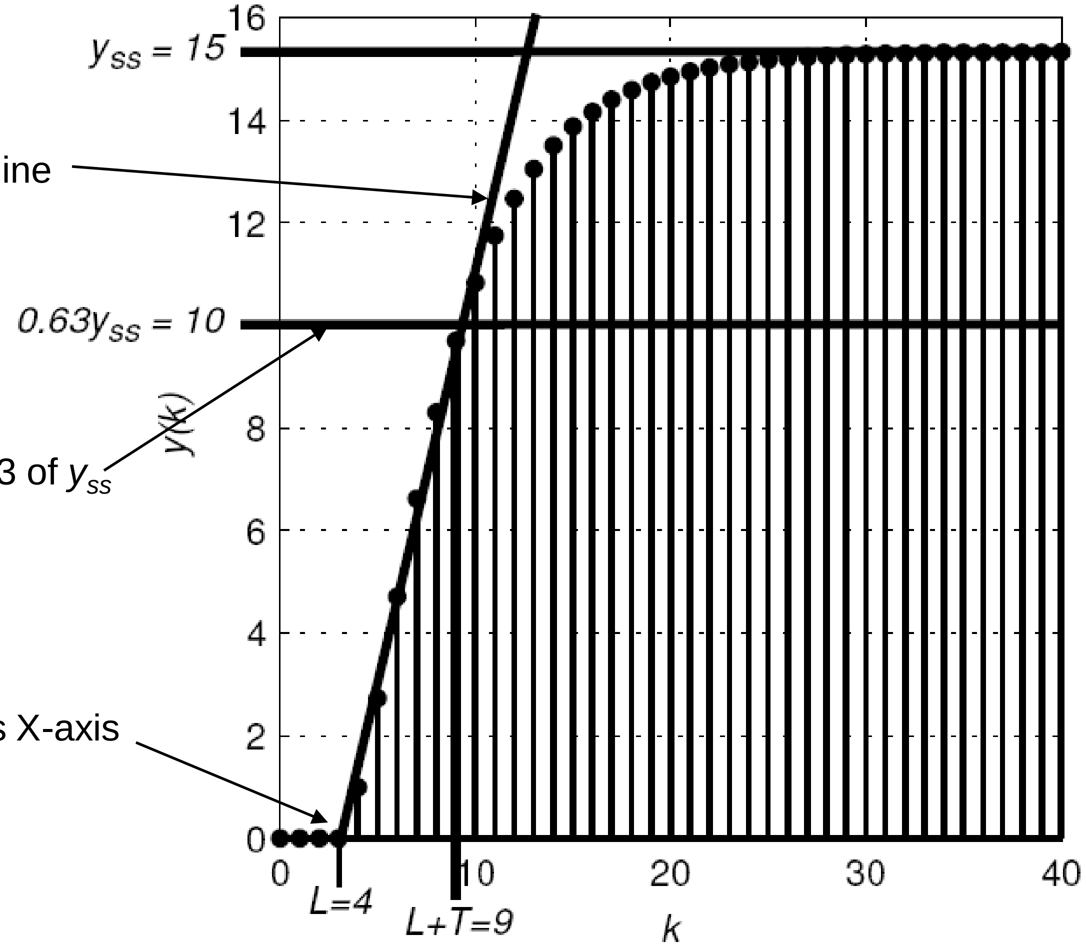
# CHR: Bump Test

Step 1: Identify Parameters L, T, R

R = Slope of tangent line

T = Time to reach ~1/3 of $y_{ss}$

L = Tangent intersects X-axis



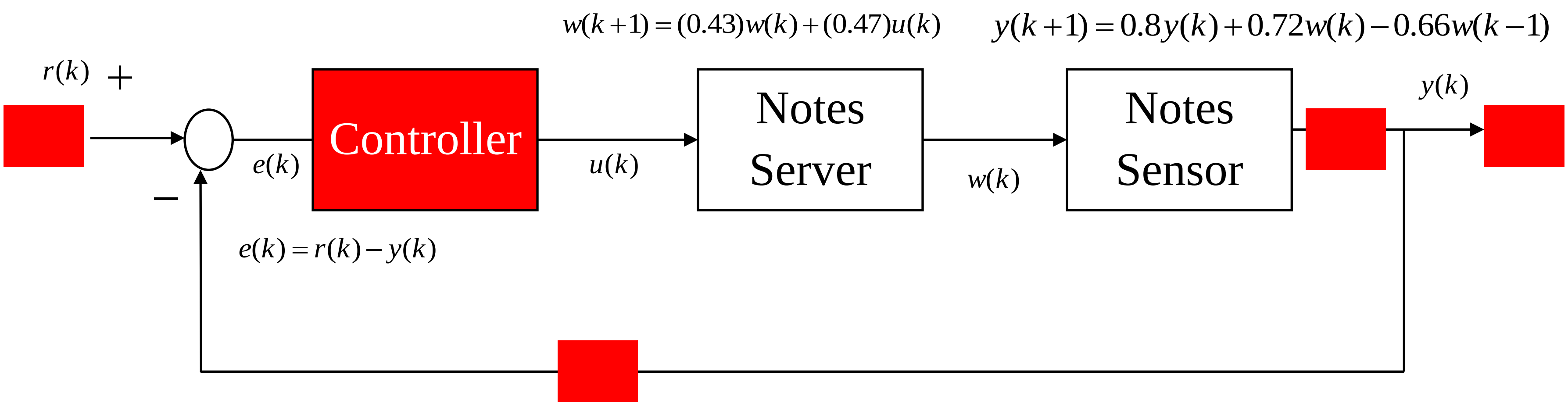

 

# CHR: Design Rules

| Design goal | Overshoot specification | Controller Gains $K_P$ | $K_I$ |
|---|---|---|---|
| Disturbance rejection | 0% | $\frac{0.6}{RL}$ | $\frac{0.15}{RL^2}$ |
| Disturbance rejection | 20% | $\frac{0.7}{RL}$ | $\frac{0.3}{RL^2}$ |
| Reference tracking | 0% | $\frac{0.35}{RL}$ | $\frac{0.3}{RLT}$ |
| Reference tracking | 20% | $\frac{0.6}{RL}$ | $\frac{0.6}{RLT}$ |

# Lab:
# Control System Analysis

# Motivating Example



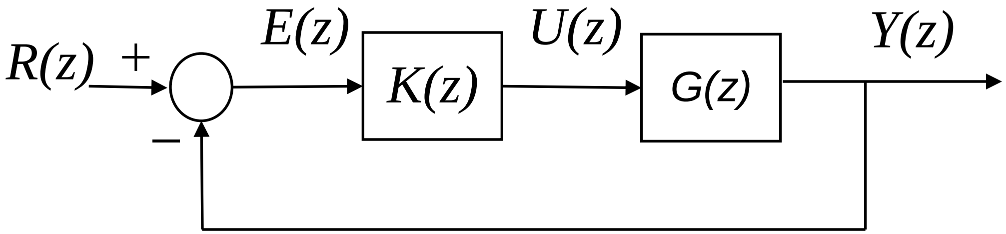

**The problem**

Design a control system that is stable, accurate, settles quickly, and has small overshoot.

Take a holistic approach

Design a control system, not just a controller

# Basic Controllers




**Proportional (P) Control**

$$u(k) = K_P e(k)$$

$$K(z) = \frac{U(z)}{E(z)} = K_P$$

**Integral (I) Control**

$$u(k+1) = u(k) + K_I e(k+1)$$

$$zU(z) = U(z) + K_I zE(z)$$

$$K(z) = K_I \frac{z}{z-1}$$

$K_P$ and $K_I$ are called ***control gains***.

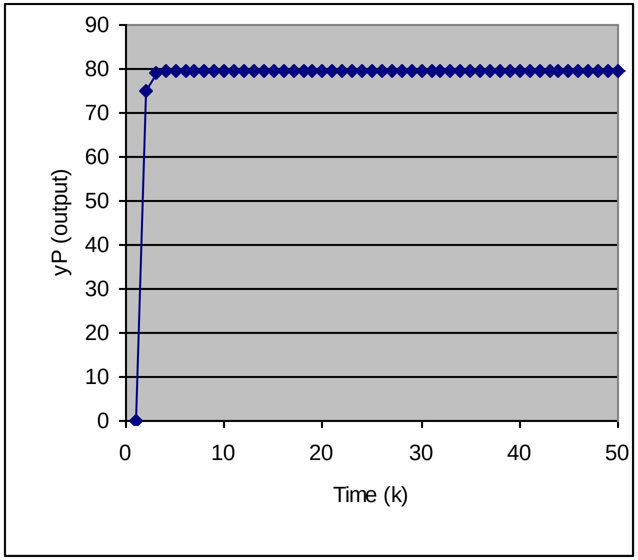
# Summary of Lab 2: P vs. I Control

**Proportional (P) Control**

$$K(z) = K_P$$

ePP(k) = r(k)-yP(k)
uP(k)=KP*eP(k)
yP(k+1)=y_coef(1)*yP(k)+y_coef(2)*uP(k)

| k | r(k) | eP(k) | uP(k) | yP(k) | KP |
|---|---|---|---|---|---|
| 0 | 200 | 200 | 160 | 0 | 0.8 |
| 1 | 200 | 124.8 | 99.84 | 75.2 | |

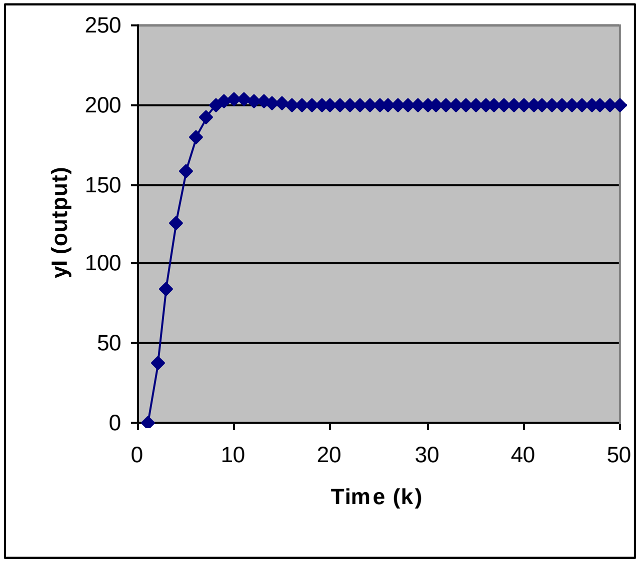



**Integral (I) Control**

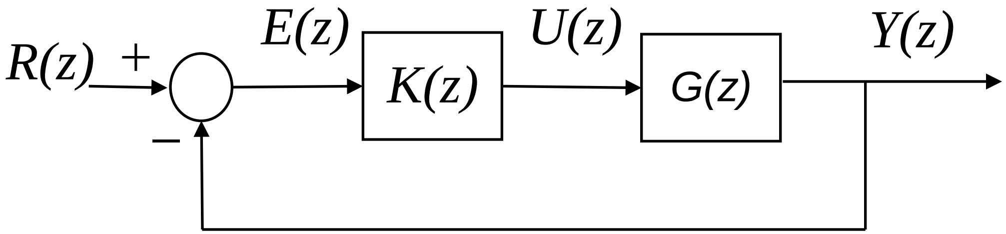
$$K(z) = \frac{K_I z}{z-1}$$

eI(k) = r(k)-yI(k)
uI(k)=uI(k-1)+KI*eI(k)
yI(k+1)=y_coef(1)*yI(k)+y_coef(2)*uI(k)

| eI(k) | uI(k) | yI(k) | KI |
|---|---|---|---|
| 200 | 80 | 0 | 0.4 |
| 162.4 | 144.96 | 37.6 | |



# Analysis




$$F_R(z) = \frac{Y(z)}{R(z)} = \frac{K(z)G(z)}{1+K(z)G(z)}$$

$$F_R^P(z) = \frac{Y(z)}{R(z)} = \frac{K_P \frac{0.47}{z-0.43}}{1+K_P \frac{0.47}{z-0.43}} = \frac{K_P}{z-0.43+0.47K_P}$$

$$p_P = 0.43 - 0.47K_P$$

$$F_R^I(z) = \frac{Y(z)}{R(z)} = \frac{K_I \frac{z}{z-1}\frac{0.47}{z-0.43}}{1+K_I \frac{z}{z-1}\frac{0.47}{z-0.43}}$$

$$= \frac{0.47K_I z}{(z-1)(z-0.43)+0.47K_I z}$$

$$= \frac{0.47K_I z}{z^2+(0.47K_I-1.43)z+0.43}$$

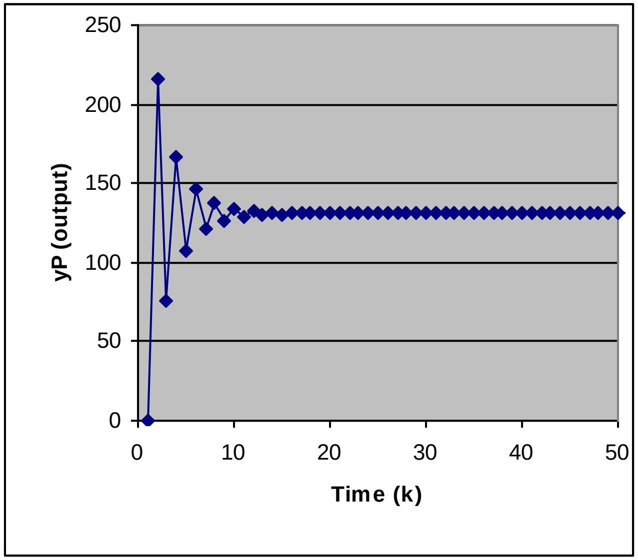
$$p_I = \frac{1.43-0.47K_I \pm \sqrt{(0.47K_I-1.43)^2-1.72}}{2}$$

## Settling Times, Steady State Gains

| Ctrl Gain | P | I |
|---|---|---|
| 0.1 | 5, 0.076 | 43, 1 |
| 0.4 | 3, 0.25 | 10, 1 |
| 3.0 | 198, 0.71 | 10, 1 |

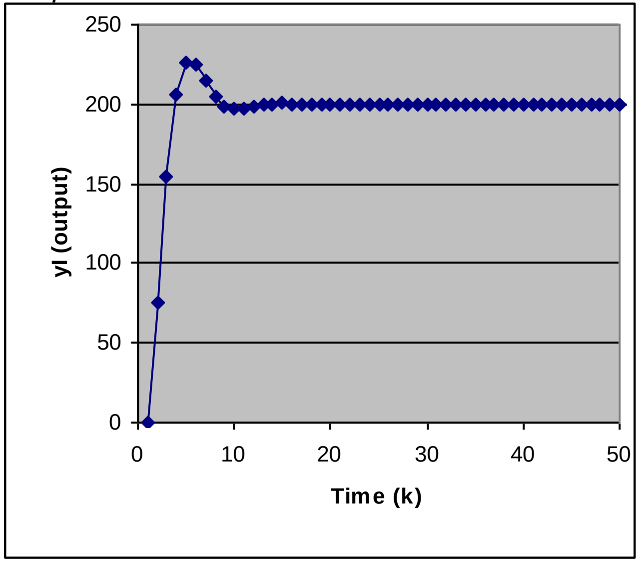
# Conclusions from P vs. I Comparison

$K_P$=2.3




$r(k)$=200

$K_I$=0.8




**<u>Conclusions:</u>**

P is fast

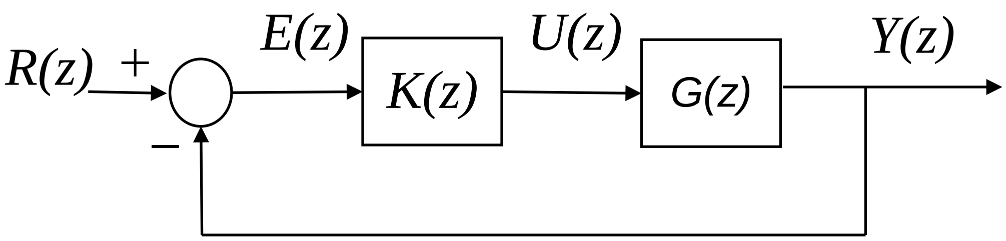
I is accurate and has less overshoot.

**<u>Design challenge:</u>**

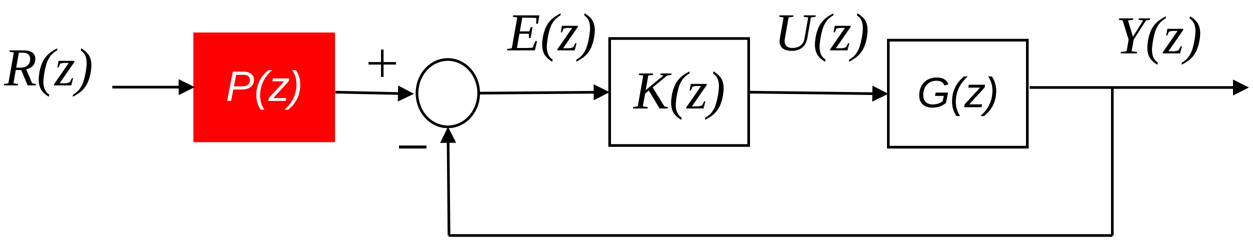
Make P accurate.

Reduce P's overshoot.

# Making P Control Accurate



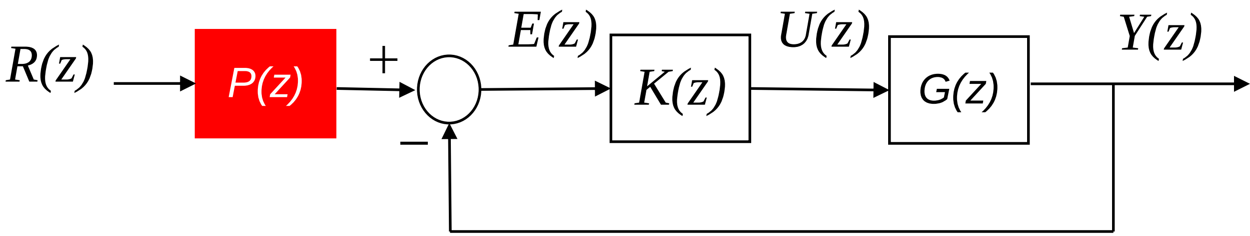

***Precompensation***: Adjusts the reference input so that the right output is obtained.




## Lab 4: Precompensation

- Modify P control to include pre-compensation
- Find a value for the precompensator that makes P control accurate
  - Trial and error
  - Adjust based on ratio between reference and output
- What happens if the reference input changes? What if the control gain changes?
- What is the general rule for the value of the precompensator?

# Computing Value of Precompensator



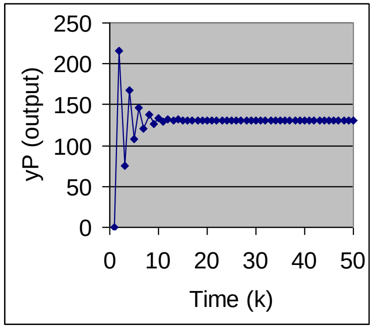

Want $R(1)P(1)F_R(1) = R(1)$

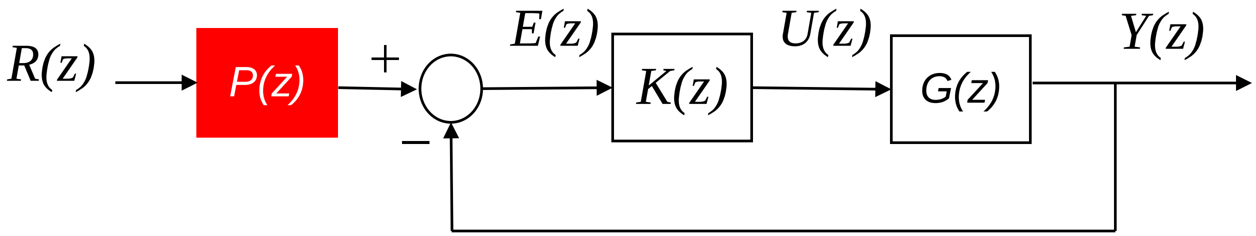
So $P(1) = \dfrac{1}{F_R(1)} = \dfrac{1 - 0.43 + 0.47K_P}{0.47K_P}$

Consider $K_P = 2.3, R(z) = 200$; then $P(z) = 1.53$

Try on spreadsheet. See if it works for other reference inputs.

# Reducing P's Overshoot

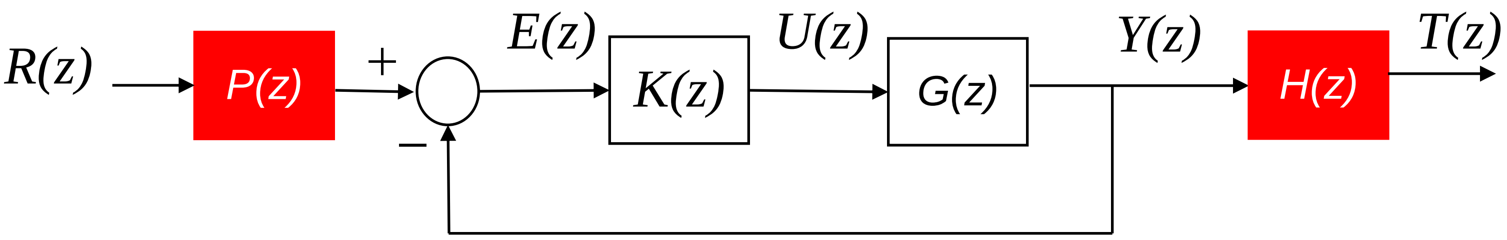



***Filter***: Smooths values over time.

c – Weight past history (make it smoother)

$t(k+1)=ct(k)+y(k+1)$

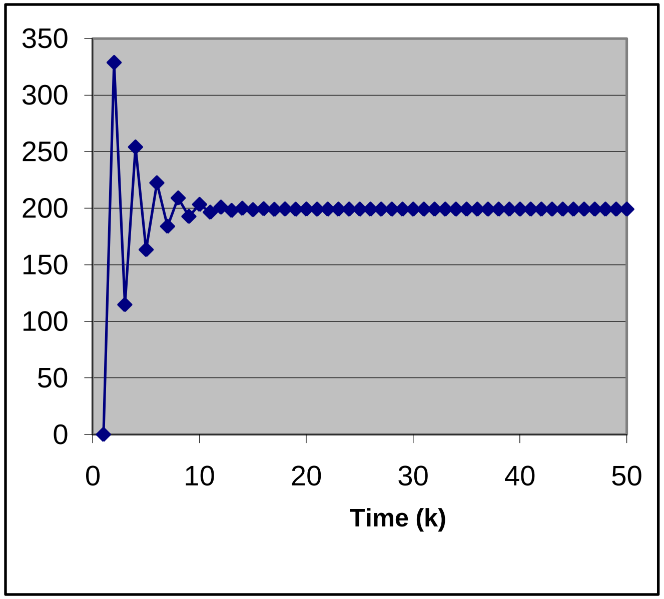



## Lab 5: Precompensation + Filter

- Add a filter to precompensated P control
- What values of *c* produce smooth *t(k)*?
- What are the other effects of the filter?

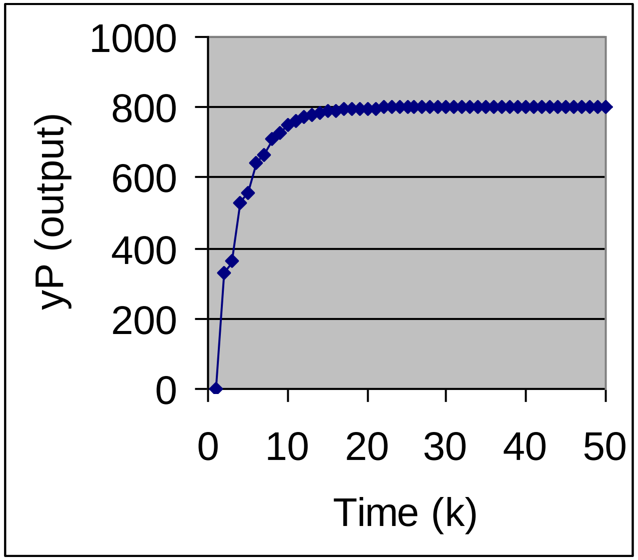
# Results of Filter Design

w/o filter




$r(k)$=200

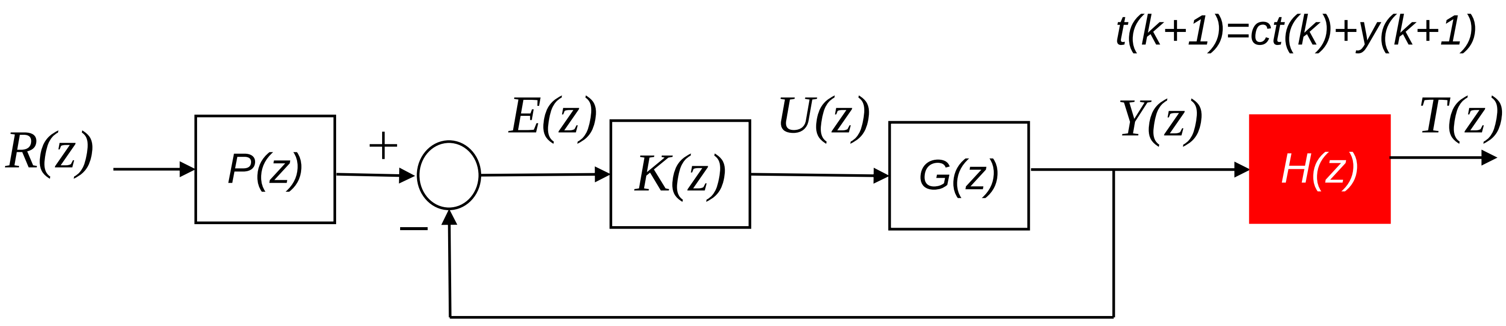
with filter: $c$ = 0.75




The good news about the filter: Can eliminate overshoot
The bad news: Inaccurate and slower.

Why inaccurate?

# Analysis of the Filter




**Analysis 1: Why does *H(z)* cause the system to be inaccurate?**

Want $P(1)F_R(1)H(1)=1$

We have designed $P(z)$ so that $P(1)F_R(1)=1$. So, it must be that $H(1)\neq 1$.

$$t(k+1)=ct(k)+y(k+1)$$

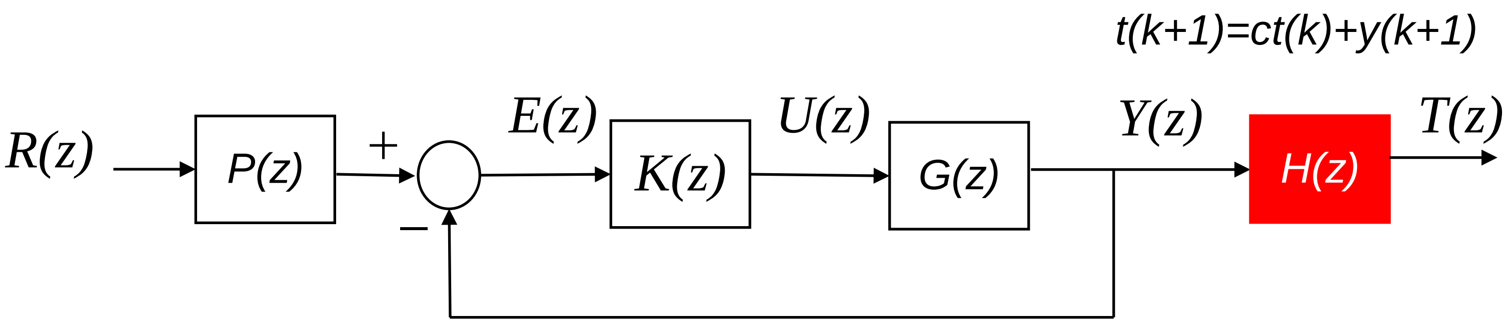
$$zT(z)=cT(z)+zY(z)$$

$$H(z)=\frac{T(z)}{Y(z)}=\frac{z}{z-c}$$

$$H(1)=\frac{1}{1-c}$$

**Check the spreadsheet.**

# Designing a Normalized Filter




Want $H(1) = 1$

Can do this by dividing by multiplying by $1 - c$.

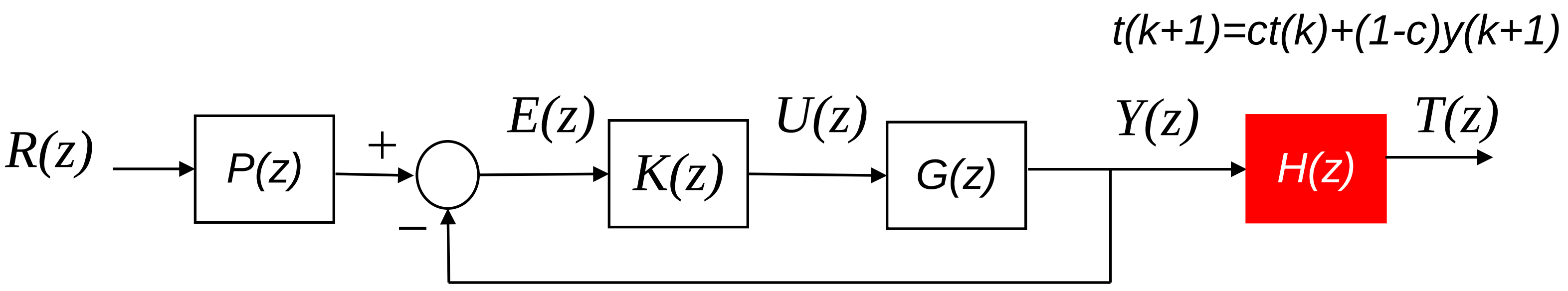
That is, use $H(z) = \dfrac{z(1-c)}{z-c}$

Converting this into a time series model, we have

$t(k+1) = ct(k) + (1-c)y(k+1)$

**Check the spreadsheet: Lab 6.**

# Analysis of the Filter




**<u>Analysis 2: Why does *H(z)* cause the system to be slower?</u>**

What are the poles of $P(z)F_R(z)H(z)$?

Let $p = \max_{poles}\{P(z), F_R(z), H(z)\}$

$P(z)$ has no poles

$$F_R(z) = \frac{0.47K_P}{z - 0.43 + 0.47K_P}$$

If $K_P = 2.3$, $F_R(z) = \dfrac{1.1}{z - 0.65}$

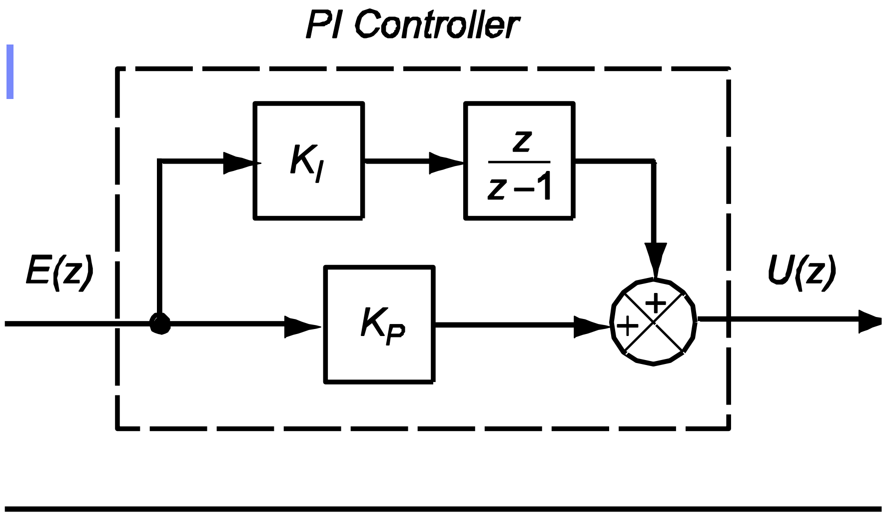
$$H(z) = \frac{T(z)}{Y(z)} = \frac{1-c}{z-c}$$

If $c = 0.75$, then there is a pole at 0.75.

So, the filter adds a closed loop pole at *c.*

**Check the spreadsheet.**

# PI Control




$$u(k) = u_P(k) + u_I(k)$$

$$K(z) = \frac{U(z)}{E(z)} = K_P(z) + K_I(z)$$

$$= K_P + \frac{K_I z}{z-1}$$

$$= \frac{(K_P + K_I)z - K_P}{z-1}$$

$$u(k) = u(k-1) + (K_P + K_I)e(k) - K_P e(k-1)$$

Lab 7: PI Control